THEORETICAL ANALYSIS ON THE PENETRATION OF POWER LINE HARMONIC RADIATION INTO THE IONOSPHERE

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ABSTRACT

In this paper we analyze theoretically the penetration characteristics of power line harmonic radiation, taking into account the presence of the anisotropic, homogeneous ionosphere. The electromagnetic field can be formulated in the wave number domain, and field distribution is obtained by taking the inverse Fourier transform. We calculate the field distribution as a function of altitude and distance from the source. It is found from the numerical results that the direct radiation from power lines is much greater than other noise, but the radiation may be amplified by inducing wave-particle interaction in the magnetosphere.

INTRODUCTION

Radiation at the fundamental and higher harmonic frequencies from power lines affects the environment of Earth's ionosphere and magnetosphere, because such radiations may lead to the enhanced electron precipitation into the ionosphere due to wave-particle interactions in the magnetosphere [1–3]. Hence, in order to estimate quantitatively the magnetospheric effect (the wave-particle interaction), we have to estimate the transmission characteristics of PLHR in the ionosphere. But there have been so far proposed very few simple theoretical computations without including the presence of the ionosphere.

In this paper we will theoretically analyze the penetration characteristics of power line harmonic radiations into the anisotropic, homogeneous ionosphere. In reality, the density of charged particles in the ionosphere has complicated profiles along altitude. Here we take the mean value of them as the zero-th order approximation. We introduce a layer approximation for the simplicity of analysis, so that we can formulate our problem by using the Fourier transform to obtain the electromagnetic fields in the wave number domain. Then, we take the inverse Fourier transform numerically to obtain the field distribution as a function of altitude and distance from the source. In order to verify the validity of the present method we compare the numerical results with the observed data.

THEORY OF PLHR PENETRATION INTO THE IONOSPHERE

In this study, we consider the fields in a relatively small region around the source. We then assume that the Earth and the ionosphere are half planes with flat boundaries. It was reported that the most possible, significant source of PLHR was zero-phase (unbalanced) harmonic currents in the power line with ground return [3]. Based on it, we can model the single electric current as the source. Thus, we consider the geometrical configuration of the present problem as shown in Fig.1.

An infinitesimally thin wire in which the current I flows is located at a height of $z = z_0$, and therefore the current density of this source is represented as $\boldsymbol{J} = \delta(x)\delta(z - z_0)I\hat{\mathbf{y}} = J_y\hat{\mathbf{y}}$, where $\delta(x)$ stands for the Dirac's delta function and $\hat{\mathbf{y}}$ is the unit vector of $+y$-direction. The Earth's magnetic field($\mathbf{H}_0$) is directed along $+z$-direction. We adopt $e^{-j\omega t}$

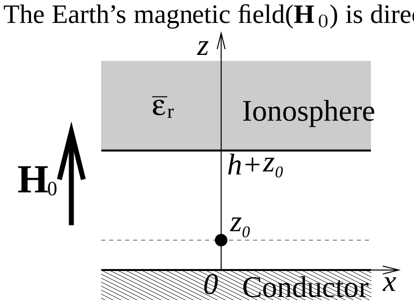


Fig. 1: Configuration for analysis

as time factor, so the permittivity tensor of plasma is given by

$$\overline{\varepsilon_r} = \varepsilon_0 \begin{bmatrix} \kappa_1 & -j\kappa_2 & 0 \\ j\kappa_2 & \kappa_1 & 0 \\ 0 & 0 & \kappa_3 \end{bmatrix}, \quad \text{where} \begin{cases} \kappa_1 &= 1 - \sum_j \dfrac{\omega_{pj}^2}{\omega^2 - \omega_{cj}^2}, \\ \kappa_2 &= \sum_j \dfrac{\omega_{cj}\omega_{pj}^2}{\omega(\omega^2 - \omega_{cj}^2)}, \\ \kappa_3 &= 1 - \sum_j \dfrac{\omega_{pj}^2}{\omega^2}. \end{cases} \tag{1}$$

and ω_{pj} and ω_{cj} are the plasma frequency and gyrofrequency of the particle species j, respectively.

The layer-configuration goes infinitely along y- and x-directions, and assuming the quasi-static current, we may let the fields be independent of y ; that is $\dfrac{\partial}{\partial y} = 0$. Thus, we introduce a pair of the Fourier transform defined as follows:

$$f(x) = \int_{-\infty}^{\infty} f(k)e^{jkx}dk, \quad \tilde{f}(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-jkx}dx. \tag{2}$$

Arranging Fourier-transformed Maxwell's equations in the plasma region yields the coupled differential equations of $\tilde{E}_z$ and $\tilde{H}_z$, the z-component of electric and magnetic fields, respectively, as follows:

$$\frac{\partial^2 \tilde{E}_z}{\partial z^2} + \frac{\kappa_1}{\kappa_3}(k_0^2\kappa_3 - k^2)\tilde{E}_z = -\omega\mu_0\frac{\kappa_2}{\kappa_3}\frac{\partial \tilde{H}_z}{\partial z}, \quad \frac{\partial^2 \tilde{H}_z}{\partial z^2} + (k_0^2\kappa_r - k^2)\tilde{H}_z = \omega\varepsilon_0\frac{\kappa_2\kappa_3}{\kappa_1}\frac{\partial \tilde{E}_z}{\partial z}, \tag{3}$$

where $k_0^2 = \omega^2\varepsilon_0\mu_0$ and $\kappa_r = \dfrac{\kappa_1^2 - \kappa_2^2}{\kappa_1}$. The other components can be expressed in terms of $\tilde{E}_z$ and $\tilde{H}_z$:

$$\tilde{E}_x = \frac{j}{\kappa_1 k}\left\{\kappa_2\omega\mu_0\tilde{H}_z + \kappa_3\frac{\partial \tilde{E}_z}{\partial z}\right\}, \quad \tilde{E}_y = \frac{\omega\mu_0}{k}\tilde{H}_z, \quad \tilde{H}_x = \frac{j}{k}\frac{\partial \tilde{H}_z}{\partial z}, \quad \tilde{H}_y = -\frac{\omega\varepsilon_0\kappa_3}{k}\tilde{E}_z. \tag{4}$$

By letting $\tilde{H}_z, \tilde{E}_z \propto e^{j\gamma z}$ the solutions to the differential equations are given by

$$\tilde{E}_z = Ae^{j\gamma_1 z} + jZBe^{j\gamma_2 z}, \quad \tilde{H}_z = -jYAe^{j\gamma_1 z} + Be^{j\gamma_2 z}, \tag{5}$$

where

$$Y = \frac{\gamma_1^2 - \frac{\kappa_1}{\kappa_3}(k_0^2\kappa_3 - k^2)}{\omega\mu_0\gamma_1\frac{\kappa_2}{\kappa_3}}, \quad Z = \frac{\gamma_2\omega\mu_0\frac{\kappa_2}{\kappa_3}}{\gamma_2^2 - \frac{\kappa_1}{\kappa_3}(k_0^2\kappa_3 - k^2)}. \tag{6}$$

and

$$\gamma_{1,2}^2 = \kappa_1 k_0^2 - \frac{1}{2}\left(1 + \frac{\kappa_1}{\kappa_3}\right)k^2 + \sqrt{w(k)} \tag{7}$$

$$w(k) = \left\{\kappa_1 k_0^2 - \frac{1}{2}\left(1 + \frac{\kappa_1}{\kappa_3}\right)k^2\right\}^2 - \frac{\kappa_1}{\kappa_3}(k_0^2\kappa_r - k^2)(k_0^2\kappa_3 - k^2). \tag{8}$$

Note that the solutions $e^{-j\gamma_1 z}$ and $e^{-j\gamma_2 z}$ are improper in Eq.(5) because no reflected wave exists in this model. For this reason, we need to take care to the singularities in taking the inverse Fourier transform.

In the air source-free region, the equations of fields are given by letting $\kappa_1 = 1, \kappa_2 = 0, \kappa_3 = 1$, then the $\tilde{E}_z$ and $\tilde{H}_z$ are uncoupled. And the source current is transformed as follows:

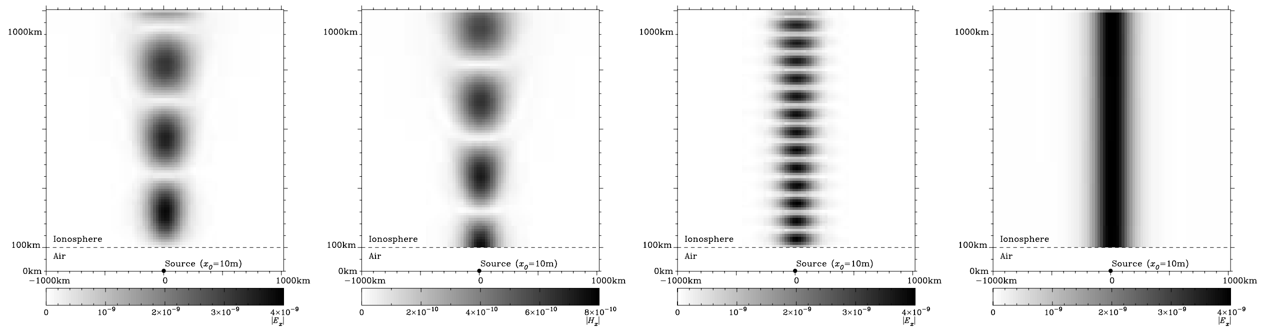
$$\tilde{J}_y = \frac{1}{2\pi}\int_{-\infty}^{\infty} J_y(x,z)e^{-jkx}dx = \frac{I}{2\pi}\delta(z - z_0). \tag{9}$$

Using usual boundary conditions that the tangential components of fields are continuous except $\tilde{H}_x$ at $z = z_0$, where $\tilde{H}_x|_{z=z_0+\epsilon} - \tilde{H}_x|_{z=z_0-\epsilon} = \int_{z_0-\epsilon}^{z_0+\epsilon} \tilde{J}_y dz = \dfrac{I}{2\pi}$, we obtain the unknown coefficients A and B. Taking the inverse Fourier transform of the field components in the wave number domain yields the ones in the space domain.

NUMERICAL RESULTS AND DISCUSSION

Based on the theory mentioned above, we can calculate the fields radiated from power lines and penetrating into the anisotropic ionosphere.

Fig.2 shows the amplitude of the field components as a function of x and z at the angular frequencies $\omega = 2\pi \times 100$,



(a) $|E_x|$ at $\omega = 100 \times 2\pi$ rad/s

(b) $|H_x|$ at $\omega = 100 \times 2\pi$ rad/s

(c) $|E_x|$ at $\omega = 200 \times 2\pi$ rad/s

(d) $|E_x|$ at $\omega = 800 \times 2\pi$ rad/s

Fig. 2: Amplitudes of field components in the ionosphere

$\omega = 2\pi \times 200$, and $\omega = 2\pi \times 800$ rad/sec which correspond to the 2nd, 4th, and 16th harmonics frequencies, respectively, of the power system in the eastern half of Japan. The used parameters are $z_0 = 10$m, $h = 1 \times 10^5$m(100km), $\mu_0 H_0 = 5 \times 10^{-5}$T, and $I = 1$A. We assumed that the plasma consists of positive ions of one kind and electrons with density $7 \times 10^{10}\text{m}^{-3}$, and that mass of the ion (hydrogen) is 1.6725×10^{-27}kg. Note that the ion gyrofrequency ω_{ci} for this case, is 4.79×10^3rad/sec (762Hz). These parameters are reasonable because in night the hydrogen ion is dominant among ions which form the ionosphere, at the altitude higher than about 600 km with the nearly constant density (about 7×10^{10} m^{-3}) [4].

From Figs.2(a) through 2(c), we can observe some peaks and valleys along the z-direction in all components, and the peak points of $|E_x|$ and $|H_x|$ are alternate in position along z-axis. And the number of the peaks increase with higher frequency. Here we omit the distribution of $|E_y|$, $|E_z|$, $|H_y|$, and $|H_z|$, but we examined that at ω =100 and 200 rad/s the peak location of $|E_y|$ and $|H_y|$ were same as $|H_x|$ and $|E_x|$, respectively, and that $|E_z|$ and $|H_z|$ were much smaller than them and negligible. We cannot say that the effect continues up to higher altitude without dissipation even though the decay is very small in the present calculation, because we now consider the homogeneous ionosphere and the reflected wave does not exist while in the real ionosphere the plasma parameters depend on altitude and the reflection exists over all region. Fig.2(d) shows the field distribution at $\omega = 2\pi \times 800$ rad/s ($f = 800$Hz), which corresponds to the first harmonic higher than the ion gyrofrequency. In contrast to the previous results, Fig.2(d) does not show any peaks and valleys along z-direction.

In order to verify the validity of our calculation, we compare the numerical results with the observed data. Here we take the observation data by [2] to compare with our numerical results. In his paper, the data observed by the satellite, Ariel4 around Winnipeg, Canada are shown in Fig.10.2.6. That figure shows the percentage occurrence of 3.2kHz signal (with 1.0kHz passband). Prominent are the data in the north hemisphere in (northern) summer and the percentage occurrence of the signal intensities is 70 and 80dB above $10^{-15}\gamma^2/\text{Hz}$ at about $330° \pm 30°$ in invariant coordinates at the observed altitude, 600 km. This angle corresponds to 2.9Mm along the azimuthal direction on the Earth's surface.

Here, we adopt the following value as parameters for computation: the height of lower boundary of ionosphere $h = 80$km, the height of power line $z_0 = 10$m, electron and ion density is $2 \times 10^{11}\text{m}^{-3}$, the mass of ion is 2.822×10^{-26}kg, and the Earth's magnetic field is 5×10^{-5}Tesla. We may simulate the situation of the diurnal ionosphere by setting the parameters as above. In day, the oxygen ion is dominant up to the altitude 1000km and the mean value of the electron and the oxygen ion and the value at the altitude 600km all are approximately $2 \times 10^{11}\text{m}^{-3}$ [4].

First, we calculate the dependence of the maximum of the magnetic flux density, Max$|\boldsymbol{B}|$, on frequency at the fixed altitude 600km which corresponds to the low altitude of the orbits of the observation satellite, Ariel 4. Fig.3 shows the numerical result of Max$|\boldsymbol{B}|$-f characteristics with the source current 1A, in terms of the field intensity above $10^{-15}\gamma^2/\text{Hz}$, where 1γ designates 1×10^{-9}Tesla. It is found that there are some peaks at about 1.9kHz and the multiples. This indicates that there exists a guided wave above this frequency, so that the fields distribute along x-direction with little damping. Actually, the integrands of the inverse Fourier integral of Eq.(5) have the poles corresponding to the guided wave modes [5].

We can find from Fig.3 that the field intensities due to power line are about 20dB on the average. It is noted that this result is for the case that the current is 1A uniformly all over frequencies. It is very difficult to estimate the harmonic current flowing into the power line as a function of frequency, and referring to [3] we adopt 0.6∼6A as the current, or

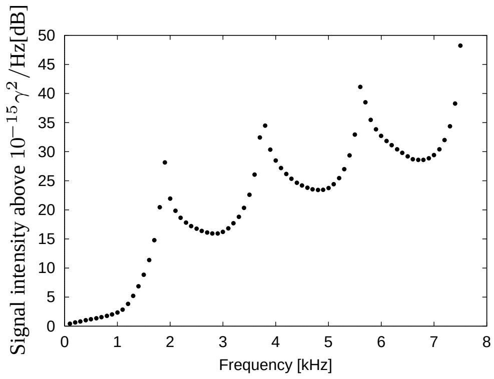


Fig. 3: The magnetic field intensity

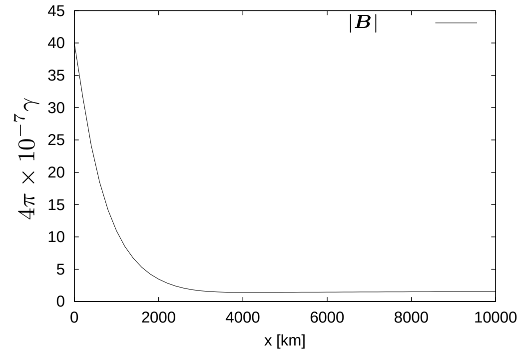


Fig. 4: $|\boldsymbol{B}|$ distribution in wider region at 3.8 kHz

adding $-4.4 \sim +16$dB to the field intensities. Moreover, the field intensities depend on the height of the source, and the dependence is approximately linear for the case of $z_0 \ll h$. Here we set $z_0 = 10$m, but several times higher lines are possible, for which we may add ~ 14dB ($\sim$ 50m). However, our calculation results do not agree with the observed one and would be much less even if the source current was estimated too high. The reasons of this disagreement is considered that the observed field is not only due to direct radiation, and it is the sum of the direct radiation and the additional amplification due to wave-particle interaction in the magnetospheric equator [1]. On the other hand, if the radiated field take the largest value estimated above, it becomes about 50dB, which is about 13dB above the Ariel4 receiver noise (37dB above $10^{-15}\gamma^2$/Hz). We can conclude that in the ionosphere the radiation from power lines may grow beyond natural and other human noises, and then induce new emissions in the magnetosphere.

In order to estimate how wide region the PLHR affects, we calculated the field distribution in a wider range up to $x = 10$ Mm at 3.8 kHz, which is shown in Fig.4. From this figure, we have estimated the region illuminated by the PLHR is within $1 \sim 1.5$ Mm, or $2 \sim 3$ Mm in both sides of the source. The power line in Winnipeg runs 500 km to northeast direction so that the affected region becomes about $\sqrt{2}$ times along azimuthal direction. With taking into account the above, our estimation with respect to the region affected by PLHR is very reasonable.

CONCLUSION

We have analyzed the penetration of power line radiation into the anisotropic ionosphere by using the layer-configuration model. The fields can be formulated by using the Fourier transform and we have calculated them by taking the numerical inverse transform. We have examined the amplitude of field components at some harmonics frequencies, and compared our numerical results with the observation by [2]. As results, we can conclude the effects of PLHR as follows: 1) The fields are directed almost horizontally. 2) The ion gyrofrequency is also a characteristic frequency to change the behavior of the field. 3) The direct radiation from power lines is much greater than other noise. However, the field observed by the satellites consists of not only the direct radiation from the power line, but also other sources, for example, due to wave-particle interaction in the magnetosphere. 4) It is verified that our estimation about the region affected by PLHR is reasonable.

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